

centerfocus output format

The result list **L** consists of points (coefficients of Poincaré differential forms) and some of their properties. These points satisfy all conditions which are required by the **centerfocus** input parameters used.

It is possible to load the **centerfocus** output file in Macaulay2.

The *Poincaré differential forms* ω treated by **centerfocus** are of the form

$$\omega := Pdx + Qdy$$

$$\text{with } P = x + p(x, y) \text{ and } Q = y + q(x, y)$$

where p and q are polynomials without constant and linear terms over a finite field F_p .

output format:

The result list **L** is structured as follows:

$$\mathbf{value}(\mathbf{L}) \quad := \quad \left\{ \begin{array}{l} \emptyset \mid \\ \text{result of experiment } 0, \\ \dots, \\ \text{result of experiment } n \end{array} \right\}$$

$$\mathbf{result\ of\ experiment\ } i \quad := \quad \left\{ \begin{array}{l} \mathit{Point}, \\ \mathit{PointID}, \\ \text{number of successive vanished focal values}, \\ \mathit{FocalValuesList}, \\ \mathit{JacobianInfo}, \\ \mathit{QuadricsInfo}, \\ \mathit{SmoothnessInfo} \end{array} \right\}$$

$$\mathbf{Point} \quad := \quad \left\{ \begin{array}{l} \text{coefficients of degree 2 monomials of the polynomial } p, \\ \text{coefficients of degree 2 monomials of the polynomial } q, \\ \text{coefficients of degree 3 monomials of the polynomial } p, \\ \text{coefficients of degree 3 monomials of the polynomial } q, \\ \dots, \\ \text{coefficients of degree } \mathbf{d} \text{ monomials of the polynomial } p, \\ \text{coefficients of degree } \mathbf{d} \text{ monomials of the polynomial } q \end{array} \right\}$$

where degree \mathbf{d} is $\mathbf{max}(\deg p, \deg q)$

'coefficients of degree k monomials of the polynomial p ' := { coefficient of polynomial p monomial x^k ,
coefficient of polynomial p monomial $x^{k-1}y$
 \dots ,
coefficient of polynomial p monomial y^k
}

PointID := { -1 or
the ID of a point entry in the centerfocus database,
see <http://87.230.76.194/centerfocus/>
}

FocalValuesList := { first focal value s_1 ,
second focal value s_2 ,
 \dots ,
 k -th focal value s_k ,
}

Length k of *FocalValuesList* is variable, and is at least

$$\min(\text{number of vanished focal values} + 1, \text{maxFocalValuesToCompute})$$

where *maxFocalValuesToCompute* is an input parameter. Maximal number of computable focal values is bounded by :

$$0 \leq \text{maxFocalValuesToCompute} \leq \frac{\text{char}(F_p) - 3}{2}$$

JacobianInfo := { fullJacobianInfo
[, subJacobianInfo] (optional)
}

fullJacobianInfo := { jacobianMatrix,
rank(jacobianMatrix)
}

jacobianMatrix is the jacobian of focal value polynomials $s_1(\dots), \dots, s_l(\dots)$ with the coefficients r_i of the polynomials p and q as function arguments. The order of function arguments used is printed at the end of the result file and is usually

$$(r_1, \dots, r_m) = (p_{20}, p_{11}, p_{02}, q_{20}, q_{11}, q_{02}, p_{30}, p_{21}, p_{12}, p_{03}, q_{30}, q_{21}, q_{12}, q_{03}, \dots)$$

where p_{ij} is the coefficient of the polynomial p monomial $x^i y^j$.
 q_{ij} is defined similarly.

$$\mathbf{jacobianMatrix} := \text{matrix} \left\{ \begin{array}{c} \left\{ \frac{\partial s_1}{\partial r_1}(\omega), \dots, \frac{\partial s_1}{\partial r_m}(\omega) \right\}, \\ \vdots \\ \left\{ \frac{\partial s_l}{\partial r_1}(\omega), \dots, \frac{\partial s_l}{\partial r_m}(\omega) \right\} \end{array} \right\}$$

where the number of rows is

$$l = \min(\text{number of vanished focal values}, \text{maxFocalValuesToCompute}).$$

and the number m of the variables r_i is $(d - 1)(d + 4)$

SubJacobianInfo is currently not used.

QuadraticsInfo will be explained in future.

some of the defined **Macaulay2** objects:

Fp	=	$\mathbb{Z}/\text{characteristic}$:	finite field
Scf	=	$F_p[\text{eps}]$:	ring of epsilon-coefficients
Rcf	=	$F_p[x, y]$:	coordinate ring of the plane
Dcf	=	$\Lambda_{F_p}[dx, dy]$:	skew commutative ring of differentials
RDcf	=	$Rcf \otimes Dcf$:	differentials with field-coefficients
SRDcf	=	$Scf \otimes Rcf \otimes Dcf$:	differentials with epsilon-coefficients

some of the defined **Macaulay2** functions:

pointDiffCF ($L\#i$)	:	get point $L\#i$ as differential form (element of SRDcf)
numberZeroValuesCF ($L\#i$)	:	number of first successive vanished focal values for $L\#i$
focalValuesListCF ($L\#i$)	:	list of computed focal values for $L\#i$ focal values are elements of the Scf ring
jacobiMatrixCF ($L\#i$)	:	jacobian matrix for focal value functions of the point $L\#i$